only a mode of flow without a core is possible. These results emerge directly from the properties of the curves Γ constructed in the axes α and β_0 for the case of grad p=const.

NOTATION

Dimensional quantities: U, velocity of upper plate; A, pressure gradient; τ_0, limiting shear stress; η_p, analog of plastic viscosity; m, n, nonlinearity parameters of flow curve; h, channel width; y_1, y_2, boundaries of core; V(y), flow velocity; $\dot{\gamma}$, shear velocity. Dimensionless quantities: W=V/U, flow velocity; ξ = y/h, vertical coordinate; ξ_1, ξ_2, boundaries of core; ξ_0, coordinate of the plane in which the shear stress equals zero; $\beta=\tau/Ah$, reduced shear stress; $\alpha=\eta_p U/(Ah)^{\frac{m}{n}}$ and $\beta_0=\tau_0/Ah$, parameters.

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STABILITY OF OPERATION OF AN APPARATUS CONTAINING A GRANULAR BED FLUIDIZED BY A GAS STREAM

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UDC 532.546

The results of numerical experiments on the investigation of the stability of the fluidization process relative to finite perturbations and its behavior upon crossing the boundary of stability are presented.

In [1] the problem of the stability of the fluidization process was formulated in a framework within which the fluidized bed was considered as a single structureless element with certain operating characteristics, and the boundary of the region of stability in the space of the parameters of the process was studied in a linear approximation.

The present report is a continuation of [1]. The stability of the fluidization process relative to finite perturbations is demonstrated by a numerical experiment and its behavior upon crossing the boundary of the region of stability is studied.

In [1] a model of a fluidized bed was proposed which is described by the following equations:

$$\frac{1}{2} m\ddot{H} + mg + k_1(q + q_v) + k_2 q = p^* - p^0, \tag{1}$$

$$q_v = cV\left(\frac{1}{2} m\dddot{H} + k_2\dot{q}\right), \tag{2}$$

$$q = q_0 + \rho S\dot{H} + \sigma\frac{H - H_0}{H}. \tag{3}$$

From the system (1)-(3) we get the equation

$$\dddot{H} + a_1\ddot{H} + (a_2 + a_3H^{-2})\dot{H} + a_4H^{-1} + a_5 = 0, \tag{4}$$

where

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$$a_0 = 2(cVmk_1)^{-1};\ a_1 = a_0\left(\frac{1}{2}m + cV\rho Sk_1k_2\right);\ a_2 = a_0\rho S(k_1 + k_2);$$

$$a_3 = 2\sigma H_0 k_2/m;\ a_4 = -a_0\sigma H_0(k_1 + k_2);\ a_5 = a_0[mg -$$

$$-p^* + p^0 + (k_1 + k_2)(q_0 + \sigma)];\ c = M/RT;\ \sigma = \rho S(Q_b - Q_0),\ q_0 = \rho SQ_0.$$

Equation (4) has the dimensionless form

$$\frac{d^3z}{dx^3} + A\left(\frac{1}{v} + \frac{Nn}{2}\right)\frac{d^2z}{dx^2} + A^2Nn\left[\frac{1}{v}\left(1 + \frac{1}{n}\right) + \frac{1}{z^2}\right]\frac{dz}{dx} -$$

$$-A^3N(1+n)\frac{1}{vz} + \frac{A}{v}\left[2B + \left(1 + \frac{1}{n}\right)(C + AND)\right] = 0, \quad (5)$$

where

$$z = \frac{H}{H_*};\ x = \left(\frac{q}{H_*}\right)^{\frac{1}{2}}t;\ \nu = \frac{H_0}{H_*^2}(Q_b - Q_0);\ A = \left(\frac{H_*}{g}\right)^{\frac{1}{2}}\nu;$$

$$B = 2\left(1 - \frac{p^* - p^0}{mg}\right);\ C = 2\frac{k_2q_0}{mg};\ D = H_*/H_0;$$

$$N = \frac{2\rho Sk_1}{\nu m};\ n = k_2/k_1;\ v = \nu ck_1V.$$

We note that N, n, and v are the dimensionless complexes which determine the boundary of the region of stability in [1].

Equation (5) was solved on a computer by the Runge–Kutta method with an accuracy of 10^{-3}. The printer put out the current values of x and z and the graph $z = z(x)$. The structural and operating parameters of two installations (laboratory and industrial), in which a transition from nonuniform fluidization to a self-oscillating mode of fluidization has been observed, were used for the calculation. The numerical values of the parameters are presented below (here and later the quantities pertaining to the industrial installation are given in brackets):

$$m = 225\,[60]\,\mathrm{kg/m^2},\ H_* = 0{,}217\,[0.118]\ \mathrm{m},\ k_1 = 387597\,[110]\,\mathrm{1/m\cdot sec},$$

$$\rho = 1.29\,[1.29]\,\mathrm{kg/m^3},\ k_2 = 3875.97\,[5.02]\,\mathrm{1/m\cdot sec},\ Q_b =$$

$$= 0.533\,[3.822]\,\mathrm{m/sec},\ S = 0.01\,[36]\,\mathrm{m^2},\ Q_0 = 0.05\,[0.65]\,\mathrm{m/sec},$$

$$H_0 = 0.15\,[0.085]\,\mathrm{m},\ p^* - p^0 = 3217.25\,[8814.55]\,\mathrm{N/m^2}.$$

The parameter V (the volume of the chamber below the grid) and accordingly the dimensionless volume v were varied in the calculations.

It was shown in [1] that the steady mode of fluidization is unstable (stable) relative to small perturbations if v lies inside (outside) the interval (v_1, v_2), where v_1 and v_2 are determined by Eq. (15) in [1] and depend on N and n. In the calculated variants v_1 and v_2 have the following values: $v_1 = 1.435$ [1.921]; $v_2 = 243.865$ [29.833].

Let us examine the behavior of the solution of Eq. (5) when v lies outside the interval (v_1, v_2), i.e., in the region of stability. It turned out that the steady mode of fluidization

$$z_* = \frac{A^2N(1+n)}{2B + \left(1 + \frac{1}{n}\right)(C + AND)} \quad (6)$$

is also stable relative to finite perturbations. Here the solution of (5) converges to z_* the faster, the farther v is from the boundary of the interval (v_1, v_2). The dependence of the dimensionless time x of establishment of an oscillation amplitude of $0.05z_*$ on the dimensionless volume v under the same initial conditions is presented in Fig. 1.

The behavior of the solution of Eq. (5) in the region of stability is shown in Fig. 2.

It is seen from Figs. 1 and 2 that operation of the fluidization process near the boundaries of the interval of instability (v_1, v_2) leads to prolonged damped oscillations upon a random departure from the steady mode z_*. If such oscillations are undesirable, then a shift of the process farther from the boundaries of (v_1, v_2) allows one to avoid them. The process can be shifted, for example, by making the volume V of the chamber below the grid smaller or larger [depending on whether our value of v is located to the left or right of (v_1, v_2)].

Upon crossing the boundary of the region of stability the breakdown of stability occurs, as suggested in [1], in a "mild" fashion. When v lies in the interval (v_1, v_2) the amplitude of the oscillations increases monoton-

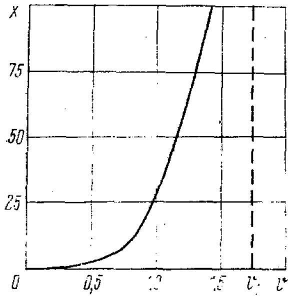

Fig. 1

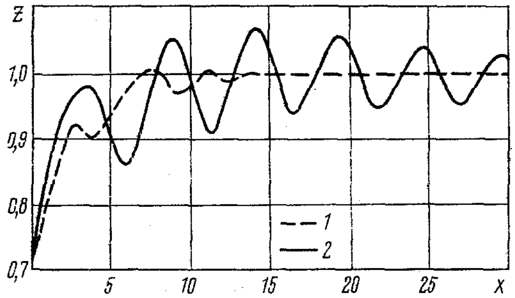

Fig. 2

Fig. 1. Dependence of dimensionless time x of establishment of a steady state on the dimensionless volume v.

Fig. 2. Behavior of the solution of Eq. (5) in the region of stability: 1) v = 0.718; 2) v = 1.108.

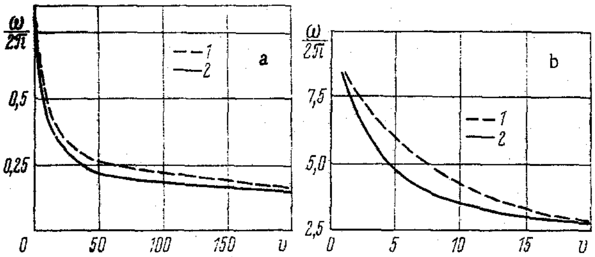

Fig. 3. Comparison of the oscillation frequency of a fluidized bed obtained from Eq. (5) (curves 1) and from the linearized system (1)-(3) (curves 2) as a function of v: a) for the laboratory installation and b) for the industrial installation.

ically to infinity the faster, the closer v is to the center of the interval (v_1, v_2). The orderly periodic oscillatory mode of fluidization shown in Fig. 4a in [1] is established only when $v = v_1$ or $v = v_2$.

The oscillation frequencies of a fluidized bed as a function of v were calculated from the linearized system (1)-(3) and from Eq. (5). The results are shown in Fig. 3.

The frequency difference does not exceed 11 [23]% and decreases near the ends of the interval (v_1, v_2). From the linearized system (1)-(3) the equation for calculating the frequency has the form

$$\omega = \left(2\,\frac{k_1 + k_2 + \nu k_1 k_2 c V}{m k_1 c V}\,\rho S\right)^{\frac{1}{2}}. \tag{7}$$

The solution of Eq. (5) in the region of instability (v_1, v_2) can be taken as correct up to the dimensionless time x_0 when z becomes equal to $z_0 = H_0/H_*$, where H_0 is the height of the bed in the motionless state. Then Eq. (5) no longer describes the physical process of fluidization. In this case the relaxation oscillations shown schematically in Fig. 4b in [1] are established.

The numerical analysis of the behavior of the solution of Eq. (5) showed that the model proposed in [1] gives a fully satisfactory criterion for the stability (instability) of operation of an apparatus containing a fluidized bed relative to finite perturbations, and also allows one to obtain the frequency of the oscillations and to estimate the damping decrement upon a perturbation of the steady mode z_*.

NOTATION

H, bed height; H_0, H_*, bed heights in motionless and steady fluidized states; g, free-fall acceleration; k_1, k_2, coefficients of resistance of gas-supply system and gas-distributing device, respectively; M, molecular weight of gas; m, mass of bed per unit cross-sectional area; p^*, p^0, pressure at inlet and outlet of apparatus; Q_0, Q_b, minimum fluidization velocity and average velocity of gas in the bubble phase; q, q_V, total mass-flow rates of gas supplied to the bed and to the free cavity; R, gas constant; S, cross-sectional area of bed; V, volume of cavity below gas-distributing grid accessible to the gas; ρ, gas density; T, absolute temperature; c, σ, q_0, parameters introduced into (4); t, time; z, dimensionless bed height; x, dimensionless time; A, B, C, D, N, n, dimensionless complexes introduced into (5); v, dimensionless volume; ν, parameter introduced into (5); z_*, dimensionless bed height in steady fluidized state; ω, circular frequency.

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EXACT SOLUTION OF COMBINED HEAT- AND MASS-TRANSFER PROBLEM DURING FILM ABSORPTION

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Exact solutions of the system of equations of heat and mass transfer accompanying absorption of vapor by a liquid film are obtained. Expressions for the main characteristics of heat and mass transfer are obtained.

Numerous processes used in chemistry, refrigeration, etc. entail the absorption of vapor by a liquid solution. A characteristic feature of such processes is the combined transfer of heat and absorbate in the liquid. In practical engineering calculations, however, heat- and mass-transfer processes are usually considered separately.

In the present paper we use a simple model to investigate the mutual effect of heat transfer and diffusion processes during absorption by a film.

The treatment of the problem of combined heat and mass transfer during absorption of a pure (with no admixture of gas) vapor by a film of solution flowing down a vertical wall is based on the following assumptions:

1) the wall is isothermal and impermeable for the absorbed substance;

2) the film thickness δ is constant;

3) the flow of liquid is laminar;

4) at the liquid—vapor interface the "absorbate—liquid solution" system is in a state of saturation;

5) wave processes in the liquid do not affect heat or mass transfer;

6) all the physical parameters of the problem (thermal diffusivity, diffusion coefficient etc.) are constant in the considered ranges of temperature and pressure.

As a model representing the state of saturation we select a linear relation between the concentration and temperature

$$\bar{C} = d\bar{T} + b.$$

The coefficients d and b are determined by the vapor pressure. We introduce a Cartesian coordinate system (x', y'), whose x' axis coincides in direction with the velocity v of liquid in the film and whose coordinate origin lies on the solid wall. We assume that in the cross section x' = 0 the liquid temperature T_0 and concentration C_0 are constant over the cross section, and C_0 is less than the saturation value corresponding to temperature T_0, i.e., $C_0 < dT_0 + b$.

We solve the problem on the assumption that v = const. In dimensionless form the system of equations representing heat and mass transfer in the film and the boundary conditions are as follows:

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